

Original Research

Online Inverse Modeling of Transient Near-Wellbore Thermal Recovery for Multirate Drill Stem Test Diagnostics after Circulation

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Abstract

Thermal disequilibrium created by drilling circulation can persist in the rock surrounding a well long after pumping has stopped, and the persistence of that disturbed zone complicates the interpretation of early drill stem test measurements. In many wells, the fluid entering the wellbore during cleanup and early flow does not originate from a thermally undisturbed formation state, but instead moves through a near-wellbore region whose temperature has been altered by prior circulation and has only partially recovered during shut-in. Because fluid properties, inflow enthalpy, and wellbore temperature response depend on that evolving thermal environment, early-time pressure and temperature diagnostics may be biased when analyzed with fixed-equilibrium assumptions. This paper develops an online inverse modeling framework for transient near-wellbore thermal recovery in circulated wells undergoing multirate drill stem testing. The framework combines radial heat diffusion in the formation, axial wellbore energy transport, compressible inflow, and temporally varying operational modes within a unified reduced-order state representation. The disturbed thermal field is treated as a latent process inferred sequentially from pressure, temperature, and rate observations, while uncertain transport and inflow parameters are updated in a regularized manner that preserves consistency with prior thermal history. The paper derives the governing equations, introduces a reduced radial-axial thermohydraulic representation, formulates a recursive estimation procedure, analyzes identifiability and stability, and outlines a field-oriented validation strategy. The resulting formulation is intended to improve the interpretation of early multirate drill stem test data in settings where the near-wellbore thermal state remains in transition and where conventional fixed-regime analyses may misrepresent entering fluid temperature and inferred flow behavior.

1. Introduction

Thermal transients around the wellbore are often treated as auxiliary effects in subsurface testing, yet in many drilling and formation-evaluation settings they directly influence the observables from which flow behavior and reservoir properties are inferred [1]. During drilling circulation, the temperature of the wellbore fluid generally differs from the native formation temperature, and the resulting thermal contrast drives heat exchange between the well and the surrounding rock. In deep or high-temperature wells this process can generate substantial cooling in the rock immediately adjacent to the wellbore [2], although the sign and magnitude of the disturbance depend on the operational sequence, fluid properties, and background geothermal conditions. Once circulation stops, the disturbed zone begins to recover toward the far-field temperature, but the recovery is diffusive and therefore slow on the scale of many operational decisions. Perforation, cleanup, and drill stem testing may occur before the disturbed rock has re-equilibrated. Under those conditions, the first fluids entering the wellbore pass through a radial thermal field that retains a memory of drilling circulation [3]. The temperatures measured during the early portion of the test, and the rate inferences drawn from those temperatures, are therefore conditioned by a hidden thermodynamic history rather than by the undisturbed reservoir state alone [4].

Table 1: Operational phases and dominant thermal behavior in the tested well

Phase	Dominant mechanism	Main thermal consequence	Data emphasis
Circulation	Axial transport with wall heat exchange	Near-wellbore rock is thermally disturbed	Temperature, pump status
Shut-in	Radial diffusion in formation	Disturbed zone recovers slowly toward far field	Bottom-hole temperature
Early inflow	Radial sampling of disturbed region	Entering fluid remains thermally biased	Temperature, pressure
Multirate flow	Coupled inflow and wellbore transport	Each rate change probes a different radial weighting	Rate, pressure, temperature
Buildup	Hydraulic relaxation with residual thermal memory	Thermal state remains continuous across segments	Pressure derivative, temperature drift

Table 2: Core state variables and their physical roles in the inverse model

Symbol/group	Type	Interpretation	Update behavior
$a_i(t)$	Thermal modes	Reduced radial coefficients of formation disturbance	Continuous, diffusion-dominated
$b_j(t)$	Wellbore modes	Axial wellbore temperature coefficients	Continuous, transport-coupled
$q_\ell(t)$	Hydraulic states	Allocation and flow-related latent variables	Faster changes near rate shifts
$\vartheta_j(t)$	Parameters	Diffusivity, heat transfer, skin or mobility scalars	Slow, regularized drift
$T_{\text{in}}(t)$	Derived latent quantity	Temperature of fluid entering the wellbore	Inferred from $a(t)$ and weighting

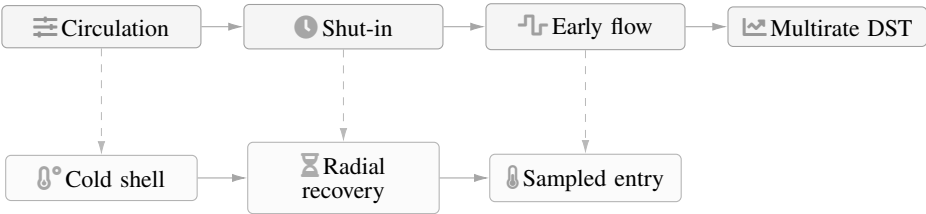


Figure 1: Operational progression from circulation to multirate testing, with the lower track indicating how the near-wellbore temperature deficit is created, relaxes during shut-in, and is then sampled by the first inflowing fluid.

This situation is more difficult than a conventional transient interpretation problem because it involves both incomplete observation and regime change. During circulation, the dominant physics are associated with axial transport in the wellbore and heat exchange across the wellbore boundary. During shut-in, the thermal field in the near-wellbore zone relaxes primarily by radial conduction, although residual wellbore equilibration may remain important. During the onset of flow, the relevant thermal quantity is not merely the wall temperature or any single radial value in the formation, but the temperature of fluid that reaches the wellbore after moving through a radially varying disturbed region. In multirate drill stem testing, each rate change occurs while the thermal field is still evolving. As a result, the mapping from hidden thermal state to measured wellbore temperature is not fixed over time [5]. Even when the operational schedule is known, the effective physical regime can lag or blur relative to those nominal

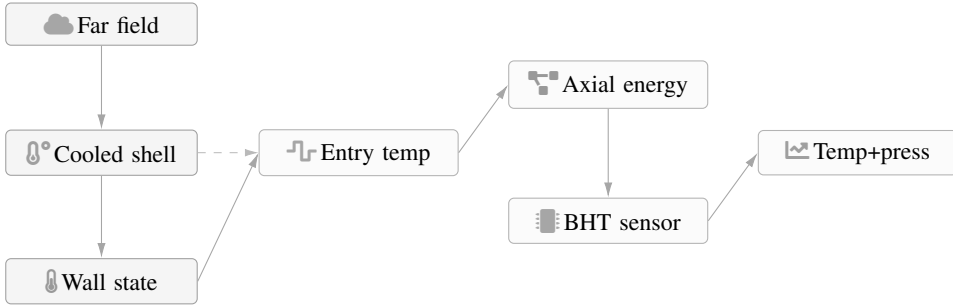


Figure 2: Reduced radial-axial coupling: the disturbed formation field near the wall determines the entering fluid temperature, which is transported upward through the wellbore energy model before appearing in the measured downhole thermal and pressure response.

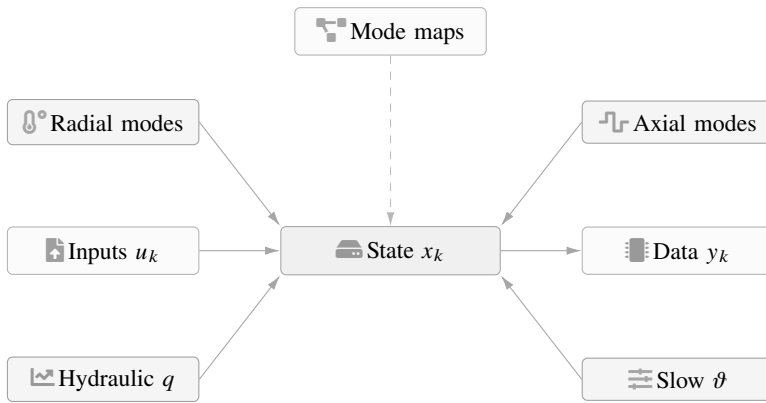


Figure 3: Mixed reduced state used by the online inverse model. Thermal modes, hydraulic allocation, and slowly drifting transport parameters are carried together so that operational transitions do not erase the inferred thermal history.

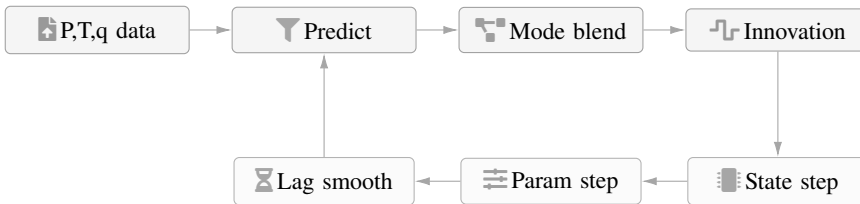


Figure 4: Recursive estimation loop for streaming drill stem test diagnostics. The online solver predicts the latent thermohydraulic state, blends mode-dependent operators, updates the state with the innovation, regularizes parameter motion, and applies short-horizon smoothing to separate thermal memory from transient mismatch.

transitions. A model that assumes a single stationary relation between state and measurement is therefore structurally inconsistent with the underlying process.

A further complication arises from the unequal timescales of hydraulic and thermal transients. Pressure often adjusts more rapidly than temperature, especially in the wellbore. A rate change can therefore generate a relatively prompt hydraulic signature while the near-wellbore thermal field remains dominated by circulation history and diffusive recovery. In practical interpretation this asymmetry matters because early-time temperature measurements may appear anomalous relative to pressure-derived expectations. If those measurements are interpreted using an equilibrium temperature assumption, inferred inflow

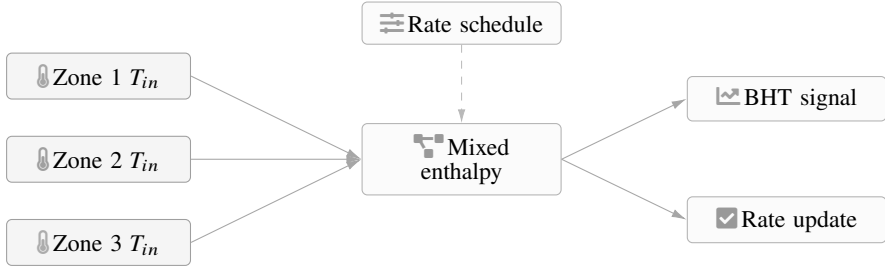


Figure 5: Multientry interpretation during rate changes. Local entering temperatures from different zones are combined into a mixed inflow enthalpy that drives the bottom-hole thermal signal and supports early-time rate updating when the near-wellbore zone is still recovering.

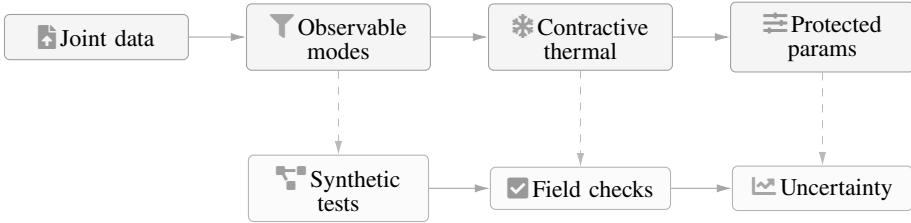


Figure 6: Identifiability and stability logic of the framework. Joint measurements determine which reduced directions are observable, dissipative thermal structure constrains online adaptation, and validation proceeds through synthetic experiments, field consistency checks, and uncertainty reporting.

Table 3: Reduced governing relations used in the thermohydraulic representation

Component	Representative relation	Role in the framework
Formation recovery	Radial diffusion of $\theta(r, z, t)$	Carries memory of circulation-induced disturbance
Wellbore energy	Axial transport of $T_w(z, t)$ with heat exchange	Maps entering enthalpy to measured temperature
Basis expansion	$\theta = \sum_i a_i \varphi_i$, $T_w = \bar{T}_w + \sum_j b_j \psi_j$	Compresses the PDE system into a tractable state
Entering temperature	$T_{in} = T_\infty + c(t)^\top a(t)$	Links radial thermal state to inflow enthalpy
Mode structure	$A_{s_t} = \bar{A} + \Delta A_{s_t}$, $C_{s_t} = \bar{C} + \Delta C_{s_t}$	Preserves physics while adapting across regimes

rate, fluid properties, or effective deliverability can be distorted. The problem is not confined to deterministic thermal correction. Because the radial temperature field in the rock is latent, the interpreter is effectively solving an inverse problem in which hidden formation temperature, wellbore heat exchange, and inflow allocation must be reconstructed sequentially from indirect data.

The persistence and practical importance of this circulation-induced thermal disturbance in the near-wellbore region were emphasized by Hasan et al. (2024), who modeled its influence on early inflow temperature and drill stem test interpretation [6]. Building on the same physical domain, the present paper develops a distinct inverse modeling perspective in which the thermal disturbance is treated as a sequentially estimated latent state and the operational transitions among circulation, shut-in, early inflow, and multirate testing are incorporated into a unified online dynamical framework [7].

The principal aim of the paper is to show that early drill stem test diagnostics after circulation can be formulated as an online reduced-order thermohydraulic inverse problem. That formulation enables

Table 4: Measurement channels and their diagnostic contribution

Measurement	Direct content	Indirect inference enabled	Typical sensitivity	
Bottom-hole temperature	Local wellbore thermal response	Entering temperature and thermal memory	High	
Bottom-hole pressure	Hydraulic transient response	Drawdown, allocation, productivity trends	High	
Surface or separator rate	Delivered flow at surface	Constraint on total inflow and mixing	Moderate	
Choke / operation flags	Scheduled control changes	Mode priors for circulation, shut-in, flow	Moderate	
Distributed wellbore temperature	Axial thermal structure	Separation of mixing and wall heat loss effects	High when available	

Table 5: Recursive estimation architecture under operational transitions

Step	Main operation	Purpose	Constraint
Prediction	Propagate $\hat{x}_{k k-1}$ with mode-weighted dynamics	Carry thermal and hydraulic history forward	Respect backbone physics
Innovation	Compute $v_k = y_k - \hat{y}_{k k-1}$	Measure mismatch between model and data	Sensor covariance aware
State update	Apply recursive gain K_k	Correct latent thermal and flow states	Maintain continuity
Parameter update	Minimize regularized loss $\mathcal{L}_k(\vartheta)$	Adjust persistent transport quantities cautiously	Slow drift only
Mode update	Refresh posterior weights $\pi_{k,\ell}$	Blend shut-in, flow, and transition behavior	Avoid abrupt switching

the incorporation of prior thermal history in a mathematically disciplined way, rather than through a disconnected sequence of stage-specific calculations. The proposed framework does not abandon the governing physics. On the contrary, it uses the physics to define the latent state, the admissible operator structure, and the consistency constraints that govern online updating. The novelty lies in the way these elements are assembled into a recursive model that can absorb new measurements while retaining physically relevant information from earlier operational phases. In this sense the method is neither a purely analytical correction nor a generic black-box predictor. It is a structured sequential estimator designed for a regime-changing physical process.

Several technical goals follow from this framing. The first is to derive a low-dimensional but physically interpretable state representation for the radial thermal disturbance around the wellbore and its coupling to axial wellbore temperature evolution. The second is to link that state explicitly to the temperature of fluid entering the wellbore during early flow, including the complications introduced by multientry inflow and rate changes. The third is to formulate a recursive parameter and state estimation problem in which uncertainty in diffusivity, effective heat-transfer coefficients, inflow allocation, and other transport quantities can be updated from data without destabilizing the underlying thermal dynamics. The fourth is to analyze identifiability and stability, since the reduced state is only partially observed and online updates can easily become nonphysical if left unconstrained. The fifth is to outline a validation protocol that focuses not just on temperature fit, but on entering fluid temperature reconstruction, early-time rate inference, and the preservation of thermal history across operational transitions [8].

The rest of the paper proceeds accordingly. The next section formulates the near-wellbore thermal disturbance and drill stem test interpretation problem as a sequential inverse problem over latent radial-axial

Table 6: Key sources of ambiguity and the model response to each

Ambiguity source	Practical effect	Mitigation in the proposed framework
Latent radial field is unobserved	Multiple thermal profiles can fit similar temperatures	Use diffusion-based reduced coordinates and history propagation
Entering temperature is not measured directly	Wellbore mixing hides inflow enthalpy	Infer T_{in} through coupled radial-axial model
Rate changes alter observability	Same state appears differently across segments	Employ mode-dependent observation operators
State-parameter trade-off	Diffusivity shifts can mimic thermal amplitude changes	Penalize parameter motion with weighted regularization
Allocation-temperature coupling	Flow partition can be confused with thermal bias	Jointly assimilate pressure, temperature, and rate data

Table 7: Multirate test interpretation quantities derived from the inferred thermal state

Quantity	Expression / source	Interpretation value	Time relevance
Local entry temperature	$T_{\text{in},k}^{(j)} = T_{\infty}^{(j)} + c_k^{(j)\top} a_k^{(j)}$	Thermal condition of inflow at each entry point	Early flow
Total entering enthalpy	$\dot{H}_{\text{in},k} = \sum_j \dot{m}_k^{(j)} h(T_{\text{in},k}^{(j)}, p_k^{(j)})$	Connects formation recovery to wellbore response	Early to mid flow
Thermal bias	$\Delta T_{\text{bias},k} = T_{\infty} - T_{\text{in},k}$	Measures deviation from equilibrium assumption	Earliest diagnostics
Rate sensitivity	$\partial \hat{T}_{bh,k} / \partial \dot{m}_k^{(j)}$	Supports thermal rate inference	Multirate segments
Allocation signature	Variation of $c_k^{(j)}$ with schedule	Reveals how different rates sample the disturbed zone	Across segment changes

states. The following section develops a reduced-order thermohydraulic model linking radial formation temperature recovery, wellbore energy transport, and multirate inflow. A subsequent section presents a recursive estimation framework with regularized parameter evolution and operational-mode transitions. The paper then analyzes identifiability, convergence, and stability, before turning to numerical realization and field-oriented experimental design. A later discussion examines practical implications for subsurface testing and thermal diagnostics. The final section summarizes the main conclusions. Throughout, the emphasis remains on a neutral technical argument: early drill stem test interpretation after circulation should account explicitly for the hidden thermal field that the operation itself has created.

2. Thermal Recovery after Circulation as a Sequential Inverse Problem

The physical system of interest is a wellbore intersecting a permeable formation whose undisturbed temperature varies with depth according to the local geothermal structure. Drilling circulation imposes a thermal forcing on the wellbore and on the formation immediately adjacent to it. The forcing may arise from cooling of a hot formation by circulating mud, from heating of a colder formation by a warmer fluid, or from more complicated axial patterns generated by well geometry and surface operating conditions. For the present discussion the most consequential case is near-wellbore cooling in a hot formation, because that is a common source of thermal bias in early drill stem testing [9]. Once circulation stops, the disturbed region around the wellbore is left in a nonequilibrium state. The temperature field in that

Table 8: Identifiability and stability considerations for online updating

Issue	Manifestation in practice	Stabilizing design choice
Weak observability of slow modes	Deep thermal recovery modes are poorly seen in short windows	Preserve history and use fixed-lag smoothing
Parameter nonuniqueness	Heat-transfer and diffusivity effects can overlap	Separate state and parameter timescales
Loss of dissipativity	Unconstrained updates may create nonphysical thermal growth	Enforce contractive thermal sub-dynamics
Transition shocks	Innovations spike near mode changes	Soft mode blending with operational priors
Weighting-vector error	Misestimated c_k corrupts entry-temperature inference	Couple thermal states with hydraulic allocation states

Table 9: Validation plan and representative performance measures

Validation stage	Target question	Example metric	Outcome focus
Synthetic recovery tests	Can the hidden thermal field be reconstructed?	Entering-temperature error $\mathcal{E}_{\text{in}}$	State fidelity
Thermal rate estimation	Does thermal information improve rate inference?	Rate error $\mathcal{E}_{\dot{m}}$	Diagnostic utility
Transition analysis	Does the model adapt without breaking history?	Adaptation score $\mathcal{A}(k^*, H)$	Consistency across modes
Field-oriented perturbation tests	How robust is the estimator to realistic data issues?	Interval coverage and bias statistics	Reliability
Ablation comparisons	Which components are essential?	Error increase after removing model blocks	Design necessity

region begins to recover diffusively, while the wellbore fluid column may continue to exchange energy with the surrounding rock. If the well is perforated and tested before equilibration is complete, the fluid entering the wellbore carries information about the disturbed formation state. The challenge is to estimate that disturbed state sequentially from incomplete measurements.

Let r denote radial distance from the well axis, z depth within the tested interval, and t time. The formation temperature is $T_f(r, z, t)$, the undisturbed formation temperature is $T_\infty(z)$, and the temperature disturbance is defined by

$$\theta(r, z, t) = T_f(r, z, t) - T_\infty(z). \quad (2.1)$$

The disturbance θ is zero in the far field and generally negative in a cooled near-wellbore zone. The quantities directly observed during the operational sequence are much more limited. Depending on instrumentation, one may measure bottom-hole temperature, bottom-hole pressure, one or more wellbore temperatures, surface or separator rate, choke settings, and operational flags identifying pump status or scheduled rate changes. None of these measurements directly reveals the radial thermal field in the rock. The field must therefore be inferred from an inverse relation linking circulation history, radial heat recovery, early inflow enthalpy, and measured temperature-pressure response.

It is convenient to view the well's life over the relevant time window as a sequence of operational modes [10]. During active circulation the formation is driven by a wellbore boundary temperature that varies in time and depth, while the wellbore temperature itself is influenced strongly by axial advection. During shut-in the circulation term disappears, and the formation disturbance recovers mainly by diffusion. During early inflow, fluid begins to move radially through the disturbed zone and enters the

wellbore at a temperature lower than the far-field formation temperature if the zone remains cooled. During multirate test flow and buildup, the forcing changes again, but the thermal state does not reset. Instead it evolves continuously from the previous mode. This continuity is what makes the problem sequential rather than taskwise. A measurement at the start of flow is not generated from a new and independent state. It is generated from the thermal field created during circulation and modified during shut-in.

The inverse problem is therefore history dependent. If one wishes to infer early inflow temperature or reconstruct the rate of fluid entry from temperature data, one must carry forward the hidden radial thermal state created earlier. A fixed-regime estimator that recalibrates only on the current window has no principled way to do that. Such a model may fit the most recent temperature data by adjusting a boundary condition or a flow coefficient, yet it does not preserve a physically meaningful representation of how much thermal deficit remains in the near-wellbore zone or how that deficit is distributed radially. This weakness becomes especially important in multirate testing, where different rate segments probe the disturbed zone differently [11]. The same average bottom-hole temperature may be compatible with different radial temperature profiles, and those profiles can imply different entering fluid temperatures once the rate changes again. The inverse problem is therefore underdetermined without a dynamical representation of the latent thermal field.

The proper mathematical statement of the problem is to estimate a hidden state x_t consisting of reduced descriptors of the radial temperature field, selected wellbore thermal variables, and uncertain transport parameters, given a sequence of measurements y_t and operational inputs u_t . The hidden state evolves under a mode-dependent transition law, while the observation law also varies with mode because the physical meaning of the measurement changes between circulation, shut-in, and flow. Thus one has a state-space structure

$$\begin{aligned} x_{t+\Delta t} &= F_{s_t}(x_t, u_t) + \xi_t \\ y_t &= H_{s_t}(x_t, u_t) + \eta_t, \end{aligned} \tag{2.2}$$

where s_t denotes the operational mode, ξ_t process discrepancy, and η_t measurement error. The role of the inverse model is to estimate x_t , possibly refine uncertain components of F_{s_t} and H_{s_t} , and propagate the information content of earlier modes into later ones.

The sequential nature of the problem has two important implications. First, the latent state should be chosen so that information about the disturbed near-wellbore zone can be transported through time with minimal loss of physical meaning. This argues for a reduced basis tied directly to radial heat diffusion rather than a generic feature representation. Second, the parameter update mechanism must be conservative in physically sensitive directions. If diffusivity or the mapping from radial field to entering temperature is allowed to change freely in response to local residuals, the model can fit recent data at the cost of destroying the thermal history that gives later predictions their physical significance. Online inverse modeling after circulation is therefore not merely a recursive filtering problem [12]. It is a structured estimation problem in which preserving the integrity of thermal history is as important as fitting the newest observations.

An additional complication arises because the entering fluid temperature itself is not observed directly in most cases. What is measured is the wellbore temperature after the entering fluid has mixed, exchanged heat with the wellbore, and undergone further transport. The inverse model must therefore estimate the entering temperature as an intermediate quantity. This makes the system doubly latent: the radial thermal field is hidden, and the inflow enthalpy generated by that field is also hidden, though it is closer to the measurement surface. The mapping from one to the other depends on rate, compressibility, entry-point configuration, and wellbore transport. Once this is recognized, the need for a coupled radial-axial model becomes evident. The next section constructs such a model in reduced-order form.

3. Reduced Radial-Axial Thermohydraulic Dynamics

The physical description begins with the formation around the wellbore. In cylindrical symmetry around a test interval, the disturbance temperature $\theta(r, z, t)$ satisfies an energy balance whose dominant term during shut-in is radial conduction. Ignoring chemical heat effects and assuming effective local thermal equilibrium between solid and fluid phases, one has

$$(\rho c)_e \frac{\partial \theta}{\partial t} = \frac{1}{r} \frac{\partial}{\partial r} \left(k_e r \frac{\partial \theta}{\partial r} \right) + q_\theta(r, z, t), \quad (3.1)$$

where $(\rho c)_e$ is the effective volumetric heat capacity, k_e is effective thermal conductivity, and q_θ represents forcing induced by the wellbore or by inflow-related advection [13]. In a cooled well, θ is negative near the wellbore after circulation and relaxes toward zero as time increases. The disturbance is generated by the boundary condition applied during circulation and by the thermal properties of the formation and wellbore. During shut-in, if wellbore heat exchange remains weak relative to formation conduction, q_θ is negligible and the recovery is diffusion dominated. During early flow, however, the production of fluid through the disturbed zone introduces an advective contribution that alters both the temperature distribution and the effective relationship between radial field and entering fluid temperature.

The wellbore is governed by a separate but coupled energy balance. Let $T_w(z, t)$ denote a representative flowing-fluid temperature in the wellbore. Then, suppressing phase transitions for clarity, the axial energy equation can be written as

$$\frac{\partial}{\partial t} (\rho_w A_w c_{p,w} T_w) + \frac{\partial}{\partial z} (\dot{m} h_w) = U(z, t) P(z) (T_f(r_w^+, z, t) - T_w(z, t)) + Q_{\text{mix}}(z, t) + Q_{\text{mech}}(z, t). \quad (3.2)$$

Here ρ_w is fluid density, A_w flow area, $c_{p,w}$ specific heat, $\dot{m}$ mass rate, h_w enthalpy, U overall heat-transfer coefficient, and P wetted perimeter. The terms Q_{mix} and Q_{mech} represent contributions from inflow mixing and mechanical work or pressure-related thermal effects. If multiple entry points exist in the tested interval, Q_{mix} is strongly influenced by the local entering temperatures and flow rates at those points. In a multirate DST, these local contributions may vary in time even when the formation properties remain fixed, because the imposed rate schedule changes the drawdown distribution and therefore the relative allocation of inflow.

To connect the latent formation field to the wellbore, a reduced radial representation is introduced. The disturbance is expanded in a basis $\{\varphi_i(r)\}_{i=1}^m$:

$$\theta(r, z, t) = \sum_{i=1}^m a_i(z, t) \varphi_i(r). \quad (3.3)$$

The basis may be chosen from diffusion eigenfunctions, finite-element shape functions on a logarithmic radial mesh, or empirically learned thermal snapshots. The essential requirement is that the basis resolve both near-wall gradients and the broader recovery envelope. In a purely diffusive setting, the basis coefficients satisfy approximately linear decay equations whose rates are governed by the eigenvalues of the radial diffusion operator. In practice, because the wall condition and the inflow condition change with operational mode, the coefficient dynamics are only piecewise diffusion dominated [14]. Nevertheless, the basis representation provides a compact and physically interpretable state.

A comparable reduction is introduced for the wellbore temperature. Let $\{\psi_j(z)\}_{j=1}^n$ be an axial basis over the tested interval and surrounding completion section. Then

$$T_w(z, t) = \bar{T}_w(z) + \sum_{j=1}^n b_j(t) \psi_j(z). \quad (3.4)$$

The full reduced state is then

$$x(t) = \begin{bmatrix} a_1(t) & \cdots & a_m(t) & b_1(t) & \cdots & b_n(t) & q_1(t) & \cdots & q_p(t) & \vartheta_1(t) \end{bmatrix}^\top, \quad (3.5)$$

where the $q_\ell(t)$ are hydraulic or inflow-allocation states and the $\vartheta_j(t)$ are slowly varying uncertain parameters such as effective diffusivity, heat-transfer coefficients, skin modifiers, or mobility scalars. This state is deliberately mixed: it contains both thermodynamic modes and uncertain coefficients because, in sequential subsurface inversion, part of what must be inferred over time is whether observed behavior is due to hidden state evolution or to uncertainty in transport parameters.

The most important hidden quantity for early flow interpretation is the temperature of fluid entering the wellbore from the formation. This entering temperature is not equal to the wall temperature in general. Nor is it equal to the far-field formation temperature. Instead it is a weighted functional of the radial temperature field sampled by the inflowing fluid. If $w(r, t)$ is a weighting kernel determined by the local inflow pattern, then

$$T_{\text{in}}(t) = T_\infty + \frac{\int_{r_w}^{r_b} w(r, t) \theta(r, t) dr}{\int_{r_w}^{r_b} w(r, t) dr}. \quad (3.6)$$

Under the reduced representation this becomes

$$T_{\text{in}}(t) = T_\infty + c(t)^\top a(t), \quad (3.7)$$

where the coefficient vector $c(t)$ depends on rate, pressure drawdown, and completion configuration [15]. Early in the flow period, $c(t)$ places more weight on modes concentrated near the wellbore. As the pressure transient penetrates outward, the weighting shifts. This time dependence is fundamental. It means that the observation operator linking latent thermal coefficients to measured wellbore temperature is itself regime dependent and time varying.

Projecting the PDE system onto the selected bases yields a reduced dynamical model. For a given operational mode s_t , one may write

$$\begin{aligned} \dot{x}(t) &= A_{s_t}(t)x(t) + B_{s_t}(t)u(t) + f_{s_t}(x(t), u(t)) + \xi(t) \\ y(t) &= C_{s_t}(t)x(t) + D_{s_t}(t)u(t) + g_{s_t}(x(t), u(t)) + \eta(t). \end{aligned} \quad (3.8)$$

The matrices $A_{s_t}(t)$ and $C_{s_t}(t)$ have different structure in circulation, shut-in, and flow. During circulation, the dominant forcing is through boundary heat exchange in the wellbore. During shut-in, the thermal part of A_{s_t} is close to a diffusion generator, and C_{s_t} maps more directly from wellbore temperature states to measured temperature. During flow, the entering-temperature functional $c(t)^\top a(t)$ becomes central and is coupled to the hydraulic states that determine inflow allocation and enthalpy transport. Because multirate testing changes rate while the thermal state remains continuous, the operator must be allowed to change with time and mode, but not in an arbitrary way.

A useful structural decomposition separates the backbone diffusion-driven dynamics from operational corrections:

$$\begin{aligned} A_{s_t}(t) &= \bar{A}(t) + \Delta A_{s_t}(t) \\ C_{s_t}(t) &= \bar{C}(t) + \Delta C_{s_t}(t). \end{aligned} \quad (3.9)$$

The backbone $\bar{A}(t)$ captures the persistent thermohydraulic structure shared across all stages, especially the reduced diffusion of the near-wellbore thermal field. The corrections $\Delta A_{s_t}(t)$ and $\Delta C_{s_t}(t)$ encode the change in state evolution and observability associated with a particular mode. This decomposition is useful both computationally and conceptually. It allows the estimator to adapt to operational shifts

while preserving the latent thermal meaning of the state. In contrast, a single unconstrained time-varying operator would fit recent data more freely but would also make the state representation unstable across time, undermining the very notion of thermal history [16].

The reduced radial-axial model thus establishes the central objects required for inverse analysis: a latent thermal state carrying the near-wellbore disturbance, a mode-dependent entering-temperature map, and a regime-sensitive but physically structured observation process. The next section formulates the recursive inverse problem built on these objects.

4. Recursive State and Parameter Estimation under Operational Transitions

Sequential estimation in this setting must address three intertwined uncertainties. The first is uncertainty in the latent thermal and hydraulic state. The second is uncertainty in transport and coupling parameters such as effective diffusivity, heat-transfer coefficients, and inflow-allocation factors. The third is uncertainty about the effective physical mode active at a given time, especially around operational transitions where the physics can be mixed. A suitable estimator should update all three, but with different degrees of flexibility. Latent states can evolve relatively quickly. Operational modes can change at scheduled times or as inferred from the data. Core thermophysical parameters should drift slowly unless sustained evidence justifies adjustment.

Discretizing the reduced model at observation times t_k , let x_k be the latent state, u_k the input, and y_k the measurement. The predictive step uses the mode-dependent model

$$x_{k+1} = A_k^{(s_k)} x_k + B_k^{(s_k)} u_k + f_k^{(s_k)}(x_k, u_k) + \xi_k, \quad (4.1)$$

with measurement relation [17]

$$y_k = C_k^{(s_k)} x_k + D_k^{(s_k)} u_k + g_k^{(s_k)}(x_k, u_k) + \eta_k. \quad (4.2)$$

Because the mode s_k may not be perfectly known physically even when the operation is scheduled, the estimator maintains mode weights rather than a single hard label. Let $\pi_{k,\ell}$ be the posterior probability of mode ℓ . The effective transition and observation operators are then blended:

$$\begin{aligned} A_k^{\text{eff}} &= \sum_{\ell} \pi_{k,\ell} A_k^{(\ell)} \\ C_k^{\text{eff}} &= \sum_{\ell} \pi_{k,\ell} C_k^{(\ell)}. \end{aligned} \quad (4.3)$$

This soft blending is particularly valuable around the onset of flow, where the thermal state is still strongly shaped by shut-in recovery even though the operation has entered a flow phase. Hard switching would force the model to reinterpret the system too abruptly.

The latent state is updated using a recursive filtering step. With a locally linearized approximation, the prediction is

$$\begin{aligned} \hat{x}_{k|k-1} &= A_{k-1}^{\text{eff}} \hat{x}_{k-1} + B_{k-1}^{\text{eff}} u_{k-1} + \hat{f}_{k-1} \\ P_{k|k-1} &= F_{k-1} P_{k-1} F_{k-1}^{\top} + Q_{k-1}, \end{aligned} \quad (4.4)$$

where F_{k-1} is the Jacobian of the predictive map and Q_{k-1} is process covariance. The update uses the innovation

$$v_k = y_k - \hat{y}_{k|k-1} \quad (4.5)$$

and the gain

$$K_k = P_{k|k-1} H_k^\top (H_k P_{k|k-1} H_k^\top + R_k)^{-1}, \quad (4.6)$$

yielding

$$\begin{aligned} \hat{x}_k &= \hat{x}_{k|k-1} + K_k v_k \\ P_k &= (I - K_k H_k) P_{k|k-1}. \end{aligned} \quad (4.7)$$

Here H_k denotes the linearized effective measurement operator. In practice, ensemble or unscented variants may be preferable when nonlinearity is strong, but the recursive structure remains the same: the state evolves continuously, while the measurement interpretation changes with mode.

The parameter update is more delicate [18]. Let ϑ_k denote the slowly varying transport and coupling parameters. A naive online update driven by immediate innovation reduction can destabilize the thermal model because several parameters affect the same outputs over short windows. To avoid that, the estimator uses a regularized incremental objective

$$\mathcal{L}_k(\vartheta) = \|v_k(\vartheta)\|_{R_k^{-1}}^2 + \lambda_\vartheta \|\vartheta - \vartheta_{k-1}\|_{W_k}^2 + \lambda_{\text{phys}} \mathcal{R}_{\text{phys}}(\vartheta), \quad (4.8)$$

where W_k is a parameter-importance metric and $\mathcal{R}_{\text{phys}}$ imposes basic admissibility, such as positivity of diffusivity and heat-transfer coefficients. The metric W_k can be interpreted as a local curvature or information matrix accumulated over previous stages. Parameters that are crucial to preserving already inferred thermal history, such as those governing the decay rates of radial thermal modes, receive stronger protection than parameters associated with local observational corrections. This differential treatment is essential. Without it, the estimator may erase previously learned aspects of the near-wellbore cooling profile when a new flow regime begins.

An important intermediate quantity in the update is the entering fluid temperature. Because it is not measured directly, the estimator treats it as an implicit latent observation tied to the formation modes. Suppose a temperature sensor near the bottom-hole records $T_{bh,k}$. The model prediction can be decomposed as

$$\hat{T}_{bh,k} = \mathcal{W}_k(\hat{T}_{\text{in},k}, \hat{b}_k, u_k), \quad (4.9)$$

where $\mathcal{W}_k$ is the wellbore transport and mixing map and $\hat{T}_{\text{in},k} = T_\infty + c_k^\top \hat{a}_k$. The innovation in bottom-hole temperature therefore constrains both the radial formation coefficients a_k and any hydraulic states influencing the entering-temperature weighting c_k . This coupling makes the sequential inverse problem more informative than one based on temperature or pressure alone, but it also makes it more ill-conditioned [19]. A small error in inflow allocation can sometimes be compensated by a change in the thermal coefficients, especially in short windows. This is why fixed-lag smoothing is helpful. By revising the recent state trajectory using later observations, the estimator can separate a true thermal-memory effect from a transient allocation error.

Mode updating is based on predictive plausibility. Given candidate modes ℓ , the posterior mode weights satisfy

$$\pi_{k,\ell} \propto \pi_{k-1,\ell}^\rho \exp\left(-\frac{1}{2} v_{k,\ell}^\top S_{k,\ell}^{-1} v_{k,\ell}\right) \Gamma_{k,\ell}, \quad (4.10)$$

where ρ is a persistence exponent, $S_{k,\ell}$ is the innovation covariance under mode ℓ , and $\Gamma_{k,\ell}$ is an operational prior incorporating scheduled changes. This mechanism allows the model to recognize that the physical state immediately after a rate change may still resemble the pre-change state more than a fully developed new regime, especially from a thermal perspective. The mode update therefore contributes

not only to classification of operational phase, but also to the correct weighting of historical thermal information.

The recursive estimation framework can be interpreted as an online inverse solver with built-in regularization against historical inconsistency. The state update absorbs new measurements. The parameter update accommodates persistent mismatch without destroying previously inferred thermal structure. The mode update governs how strongly the current measurement should be interpreted through the lens of circulation, shut-in, or flow dynamics. Together, these components transform the well-testing sequence into a single evolving inverse problem rather than a disconnected series of separate fits.

5. Multirate Drill Stem Test Interpretation from the Inferred Thermal State

The reduced inverse model becomes most valuable once the well enters the test phase, because that is where the hidden near-wellbore thermal field affects practical diagnostic quantities [20]. In multirate drill stem testing, one seeks to infer flow behavior, effective entry conditions, and reservoir-relevant signatures from a sequence of flow and buildup periods. If the near-wellbore zone remains thermally disturbed, then the temperature associated with early inflow depends on the radial recovery state, and that in turn affects the wellbore temperature response and any temperature-based flow interpretation. A model that estimates the hidden thermal state can therefore support more consistent inference than one that assumes an undisturbed entry temperature.

Consider a test interval with N_p possible inflow points or grouped entry zones. For entry point j , let the local radial thermal coefficients be $a_k^{(j)}$, the local inflow weighting vector be $c_k^{(j)}$, and the local entering temperature be

$$T_{\text{in},k}^{(j)} = T_{\infty}^{(j)} + c_k^{(j)\top} a_k^{(j)}. \quad (5.1)$$

Let $\dot{m}_k^{(j)}$ be the local mass inflow rate, so that

$$\sum_{j=1}^{N_p} \dot{m}_k^{(j)} = \dot{m}_{\text{tot},k}. \quad (5.2)$$

The enthalpy entering the wellbore is then the weighted sum of the local inflow enthalpies. If the flowing phase is approximated by a temperature-dependent enthalpy law $h(T, p)$, the total entering enthalpy at the bottom-hole is

$$\dot{H}_{\text{in},k} = \sum_{j=1}^{N_p} \dot{m}_k^{(j)} h\left(T_{\text{in},k}^{(j)}, p_k^{(j)}\right). \quad (5.3)$$

This entering enthalpy feeds the wellbore temperature evolution and can substantially influence the bottom-hole thermal response, especially early in a flow period when the disturbed near-wellbore zone is sampled most strongly.

A common practical task is to infer rate from temperature when direct rate measurement is delayed, uncertain, or only available at the surface. The inverse model supports this by exploiting the sensitivity of the wellbore temperature to both entering enthalpy and local allocation. Denote the predicted bottom-hole temperature by $\hat{T}_{bh,k}$. Its sensitivity to local inflow rate is

$$\frac{\partial \hat{T}_{bh,k}}{\partial \dot{m}_k^{(j)}} = \frac{\partial \hat{T}_{bh,k}}{\partial h_{\text{in},k}^{(j)}} \frac{\partial h_{\text{in},k}^{(j)}}{\partial \dot{m}_k^{(j)}} + \frac{\partial \hat{T}_{bh,k}}{\partial T_{\text{in},k}^{(j)}} \frac{\partial T_{\text{in},k}^{(j)}}{\partial \dot{m}_k^{(j)}}. \quad (5.4)$$

The second term is particularly important in a thermally disturbed well because the entering temperature itself depends on the drawdown-induced weighting across the radial thermal field. Thus temperature does not merely reveal the enthalpy of a fixed inflow stream [21]. It reveals how the current rate schedule is sampling the disturbed formation. If that dependence is ignored, the rate inferred from early temperature data can be biased even when the pressure data appear reasonable.

The multirate structure of a DST creates an additional inverse benefit. Different rate levels illuminate different portions of the radial thermal disturbance. A low initial flow rate may sample a narrow region near the wellbore and remain strongly influenced by the cooled zone. A later higher rate may increase the effective radial weighting and admit more contribution from warmer formation farther from the well. If the hidden thermal state is estimated consistently across these changes, then the multirate schedule becomes an information source rather than only a complication. The sequential inverse model can use the changes in temperature response across rate levels to refine the radial thermal coefficients and thus improve the estimate of both current entry temperature and remaining thermal recovery.

To formalize this, let u_k include the rate schedule and let the mode variable s_k distinguish the current segment of the test. The observation model for temperature and pressure during segment ℓ may be written as

$$y_k^{(\ell)} = \begin{bmatrix} T_{bh,k} \\ p_{bh,k} \end{bmatrix} = \mathcal{G}^{(\ell)}(x_k, u_k) + \eta_k. \quad (5.5)$$

When the segment changes from ℓ to $\ell + 1$, the hidden state x_k remains continuous, but the map $\mathcal{G}^{(\ell)}$ changes because the rate and therefore the inflow weighting and wellbore transport have changed. This is precisely why a mode-aware estimator is required. If the model were to absorb the resulting innovation only by changing the hidden state, it would distort the inferred thermal history. If it were to absorb the innovation only by changing the observation map without continuity of the state, it would discard information about the radial disturbance [22]. The joint treatment preserves both.

Another diagnostic quantity of interest is the apparent equilibrium inflow temperature that would be inferred incorrectly if the disturbed zone were ignored. Let

$$\Delta T_{\text{bias},k} = T_{\infty} - T_{\text{in},k}. \quad (5.6)$$

This quantity measures the thermal deficit at the point of entry and can be propagated into property estimates. For example, if density or viscosity depends on temperature, then the relative property bias induced by ignoring $\Delta T_{\text{bias},k}$ can be approximated by

$$\begin{aligned} \Delta \mu_k &\approx \left. \frac{\partial \mu}{\partial T} \right|_{T=T_{\infty}} \Delta T_{\text{bias},k} \\ \Delta \rho_k &\approx \left. \frac{\partial \rho}{\partial T} \right|_{T=T_{\infty}} \Delta T_{\text{bias},k}. \end{aligned} \quad (5.7)$$

In gas or volatile systems, even moderate temperature bias can alter compressibility and phase behavior sufficiently to change inferred flow rates or effective productivity. Thus the hidden thermal state estimated by the inverse model is not only a temperature correction. It is an intermediate variable with direct implications for fluid-property interpretation.

The framework also clarifies why rate changes may affect temperature for longer than pressure. Pressure transients react promptly to a change in drawdown, whereas the thermal contribution from the near-wellbore zone continues to reflect the previously disturbed radial field. The wellbore temperature may therefore exhibit delayed convergence toward the new flow regime, even when the pressure derivative appears settled. In a conventional interpretation this can look like a nuisance transient or a sensor lag. In the reduced inverse model it appears naturally as the finite-time response of the hidden thermal state through the entering-temperature functional and the axial wellbore energy balance.

The practical implication is that temperature should not be treated as a passive corroborative measurement during early multirate DST interpretation [23]. It is an active source of information about the current state of near-wellbore thermal recovery. When coupled with pressure and rate data through an appropriate inverse model, it can improve estimation of entry temperature, allocation, and early inflow behavior. The next section examines under what theoretical conditions that inference remains stable and identifiable.

6. Identifiability, Stability, and Error Propagation

A reduced inverse model for near-wellbore thermal recovery is valuable only if its hidden states and parameters are sufficiently identifiable from the available data and if its online updates do not introduce instability or nonphysical thermal behavior. These two issues are closely linked. If the data weakly constrain a parameter or a mode, unrestricted updating in that direction can create unstable dynamics while appearing to improve immediate fit. Conversely, excessive rigidity can suppress legitimate adaptation at operational transitions. The analysis in this section clarifies the structure of that trade-off.

Begin with the latent-state observability problem. Consider the linearized reduced system over a finite horizon H :

$$\begin{aligned} x_{k+1} &= A_k x_k + \xi_k \\ y_k &= C_k x_k + \eta_k, \end{aligned} \quad (6.1)$$

where A_k and C_k denote the effective mode-weighted operators along the current trajectory. The empirical observability matrix over horizon H is [24]

$$O_{k,H} = \begin{bmatrix} C_k \\ C_{k+1} A_k \\ C_{k+2} A_{k+1} A_k \\ \vdots \\ C_{k+H-1} A_{k+H-2} \cdots A_k \end{bmatrix}. \quad (6.2)$$

If the smallest singular value of $O_{k,H}$ is close to zero, then certain latent directions are poorly observable over that horizon. In the present setting, the fast near-wall thermal modes may be highly observable during early flow because they influence T_{in} strongly, but the broader recovery modes may be weakly observable unless the shut-in or later temperature data are sufficiently informative. Likewise, some hydraulic allocation states may be weakly distinguishable from thermal states if only a single wellbore temperature is available. This pattern of observability is not static. It changes with mode. That is why the online estimator needs continuity of state and regularized parameter motion: information obtained in one mode may be essential later when that same direction becomes weakly observed.

The most important identifiability issue concerns the separation of radial thermal state from uncertain transport parameters. Changes in effective diffusivity, for example, can mimic changes in the amplitude of slow thermal modes over a short window. Changes in the wellbore heat-transfer coefficient can mimic changes in entering temperature if only one downhole temperature sensor is available. The recursive estimator mitigates this by letting state and parameter evolve on different timescales and by penalizing parameter motion according to an importance metric. In mathematical terms, if the incremental parameter update $\delta \vartheta_k$ is chosen to minimize a regularized local loss, then the solution satisfies

$$\delta \vartheta_k = - \left(J_k^\top R_k^{-1} J_k + \lambda_\vartheta W_k \right)^{-1} J_k^\top R_k^{-1} \nu_k, \quad (6.3)$$

where J_k is the Jacobian of the prediction with respect to parameters. The term $\lambda_\vartheta W_k$ shrinks updates in directions that are either physically sensitive or historically important, thereby reducing the risk that

current data are explained by inappropriate parameter shifts. This does not remove nonidentifiability, but it makes the ambiguity operationally manageable.

Stability of the thermal subdynamics requires special care. The reduced radial modes should inherit the dissipative character of heat diffusion [25]. Let a_k denote the vector of formation thermal coefficients and suppose the linearized update is

$$a_{k+1} = A_k^{(a)} a_k + \chi_k, \quad (6.4)$$

where χ_k represents forcing and coupling terms. In a pure recovery phase, the nominal operator should be contractive in a suitable norm. A natural condition is the existence of a positive definite matrix P_a such that

$$A_k^{(a)\top} P_a A_k^{(a)} - P_a \preceq -Q_a, \quad (6.5)$$

for some $Q_a \succ 0$, at least when external forcing is absent. This inequality expresses discrete dissipativity of the reduced thermal modes. It implies that a quadratic thermal energy measure decays over time in the absence of new forcing. If online updating of the operator is unconstrained, this property can be lost. A model may then fit recent temperature data by inventing oscillatory or amplifying thermal modes that have no physical basis. The stability regularization therefore acts directly on the thermal subblock, shaping its spectrum and its alignment with the nominal diffusion operator.

The interaction between state-estimation error and parameter adaptation can be described by the error recursion

$$e_{k+1} = F_k e_k + G_k \delta \vartheta_k + \zeta_k, \quad (6.6)$$

where e_k is state-estimation error, F_k is the filtered state transition, G_k maps parameter-update error into state error, and ζ_k collects process and measurement contributions [26]. If the operator norm of F_k is near one and $G_k \delta \vartheta_k$ is not controlled, then even small parameter updates can accumulate into large thermal-state error. This phenomenon is especially problematic immediately after mode changes, because the innovation is largest there and the temptation to adapt parameters aggressively is strongest. The regularized update scheme counters this by letting the mode weights absorb part of the innovation and by restricting parameter movement until persistent evidence accumulates.

Error in the entering-temperature estimate has particularly strong downstream consequences. Let the entering-temperature estimate be $\hat{T}_{\text{in},k} = T_\infty + c_k^\top \hat{a}_k$. Then

$$T_{\text{in},k} - \hat{T}_{\text{in},k} = c_k^\top e_k^{(a)} + (c_k - c_k^\star)^\top a_k^\star, \quad (6.7)$$

where $e_k^{(a)}$ is the thermal-state error and $c_k^\star$ is the true inflow-weighting vector. The first term arises from error in the thermal field; the second arises from error in the hydraulic weighting that maps the field to entry temperature. The second term is often overlooked in simpler corrections. Yet in multirate testing it can be significant because the pressure and mobility conditions defining c_k change with the schedule. A stable and interpretable model must therefore control not only the state error but also the consistency of the inflow-weighting estimate.

There is also a coordinate issue. The reduced basis representation anchors the latent state physically, but some residual freedom remains because basis scaling or weakly observed linear combinations can still vary. To preserve temporal comparability, the model keeps the basis fixed and updates only the state coordinates and operator coefficients. This is a deliberate choice [27]. If the basis were allowed to drift online, memory of earlier thermal states would become difficult to interpret, and the inverse problem would acquire a severe nonidentifiability not justified by the data. The fixed-basis design sacrifices some flexibility but preserves the meaning of thermal history, which is central to the operational problem.

Overall, the analysis shows that identifiability in this application is local, mode dependent, and strongest when temperature, pressure, and rate data are interpreted jointly. Stability relies on respecting

the dissipative structure of the thermal subdynamics and on controlling parameter updates in directions with strong influence on the thermal history. These findings justify the regularized, mode-aware recursive design introduced earlier and inform the experimental protocol developed next.

7. Numerical Realization and Field-Oriented Validation Design

A practical implementation of the proposed framework should respect the computational realities of well-test analysis while retaining enough structure to represent thermal memory faithfully. The numerical realization begins with an offline model-reduction stage. Synthetic or semi-analytical solutions of the radial thermal diffusion problem are generated over a range of circulation durations, wellbore thermal forcings, shut-in times, and flow conditions. These solutions provide snapshots of $\theta(r, t)$ over the relevant radial interval. A reduced basis is then extracted using either a diffusion-eigenfunction library or a proper orthogonal decomposition weighted by the radial measure $r \, dr$. The resulting basis should resolve steep near-wall gradients as well as the broader recovery profile. A similar basis is constructed for the wellbore temperature field over the tested interval and any adjoining completion section whose thermal response materially affects the measured bottom-hole temperature.

The reduced-order operators can then be assembled by projection of the governing equations and by inclusion of parametric dependence on uncertain transport quantities [28]. In practice, it is useful to partition the reduced state into a small number of fast thermal modes, a small number of slow thermal modes, a compact wellbore thermal substate, and a low-dimensional hydraulic substate. This partition improves interpretability and makes it possible to tune regularization differently for each block. Fast near-wall modes, for example, should be allowed to evolve rapidly under early inflow, but their decay rates should remain physically constrained. Slow modes should change even more conservatively. Hydraulic allocation states may need greater agility around rate changes. The block structure of the reduced model supports such differentiated treatment.

Online computation is organized into prediction, correction, mode update, and parameter update. Because the observation vector is typically small, the recursive update is inexpensive. The dominant cost lies in maintaining the local Jacobians needed for parameter adaptation and in evaluating the reduced wellbore transport map when multiple entry points are active. Even there, the cost remains modest because the latent dimension is small compared with any full radial-axial discretization. This is an advantage over direct simulation-based inverse methods, which can be too expensive for real-time or quasi-real-time interpretation during a test. The reduced inverse model is not meant to replace detailed simulation entirely, but to make online thermally informed interpretation feasible within operational time constraints [29].

Validation should be conducted in stages. A first stage uses fully synthetic cases in which the true radial thermal field, entering temperatures, and inflow allocations are known. These cases should vary circulation duration, thermal contrast between drilling fluid and formation, effective diffusivity, shut-in duration, and multirate schedule. The purpose is to establish whether the estimator can reconstruct the hidden radial field and the entering fluid temperature from the observable temperature-pressure signals. A key metric is the entering-temperature error,

$$\mathcal{E}_{\text{in}} = \frac{1}{N_f} \sum_{k \in \mathcal{I}_{\text{flow}}} \left(T_{\text{in},k}^* - \hat{T}_{\text{in},k} \right)^2, \quad (7.1)$$

where $\mathcal{I}_{\text{flow}}$ indexes flow observations. Because early-time behavior is the main operational target, a weighted variant that emphasizes the first part of each flow segment is also useful.

A second stage should evaluate rate reconstruction from thermal information. If the synthetic truth provides local and total inflow rates, then the estimator's rate error can be measured by

$$\mathcal{E}_{\dot{m}} = \frac{1}{N_f} \sum_{k \in \mathcal{I}_{\text{flow}}} \left(\dot{m}_{\text{tot},k}^* - \hat{m}_{\text{tot},k} \right)^2. \quad (7.2)$$

It is also valuable to record how much of any improvement in rate estimation is attributable specifically to the thermal part of the model. This can be done by comparing the full estimator against a reduced variant in which the entering temperature is fixed at the undisturbed formation temperature. The difference between the two reveals the practical significance of honoring the disturbed near-wellbore zone.

A third stage focuses on operational transitions [30]. Here the goal is not only low error, but good adaptation without loss of consistency. For a transition starting at time index k^* , one may define an adaptation metric

$$\mathcal{A}(k^*, H) = \frac{1}{H} \sum_{j=0}^{H-1} \ell(y_{k^*+j}, \hat{y}_{k^*+j|k^*-1}), \quad (7.3)$$

where ℓ is an application-relevant loss. Low values indicate rapid accommodation of the new mode. Yet adaptation alone is insufficient. One must also verify that the inferred thermal state remains consistent with the earlier history. This can be checked by comparing the reconstructed thermal modes just after a transition with the expected continuation of the pre-transition state under the physical model. Excessively large discontinuities in those modes are a sign that the model is fitting local residuals at the cost of thermal consistency.

A fourth stage involves semi-synthetic or field-oriented scenarios with imperfect data. Measurement gaps, sensor bias, delayed rate data, and uncertain operational timing should all be introduced because they are common in practice. Under these conditions the estimator's uncertainty quantification matters as much as its point predictions. Predictive interval coverage for temperature and pressure should be reported, along with uncertainty in entering temperature and rate estimates. The model should become less confident during ambiguous transitions and regain confidence as more data accumulate [31]. Overconfidence is particularly harmful here because it can make an incorrect thermal correction look deceptively precise.

Field validation is necessarily less direct because the true radial thermal field is not observed. Nevertheless, useful criteria remain available. The model can be assessed by its ability to match measured downhole temperature and pressure with physically plausible parameters, by the consistency of its inferred entering temperature with known thermal history, and by the degree to which its early-time rate estimates reconcile temperature and surface measurements. Comparison with simpler thermal-correction methods or fixed-regime models is especially informative. If the proposed model yields a materially different entering temperature or early rate, the analyst can examine whether that difference corresponds to a more plausible thermal recovery trajectory given the circulation history and shut-in time.

Finally, ablation studies should test the necessity of the major modeling choices. Removing the reduced radial thermal state and replacing it with a single corrected temperature should reveal whether the hidden field really matters. Removing mode-dependent observation updates should show whether a fixed observation map is inadequate across operational transitions. Removing parameter regularization should expose the extent to which unconstrained online updating destabilizes the thermal interpretation. These ablations are valuable not only methodologically, but also operationally, because they indicate which parts of the framework are essential for field deployment and which might be simplified without major loss.

8. Discussion

The inverse formulation developed in this paper suggests a different way of thinking about post-circulation drill stem testing [32]. Rather than treating circulation-induced near-wellbore cooling as a preliminary correction applied before normal test interpretation begins, the framework treats the cooling and recovery process as a continuing part of the state of the well during the test itself. This point may appear subtle, yet it changes how the measurement sequence is understood. If the disturbed thermal field is part of the latent state, then every new temperature observation during the early test phase carries information not only about the current rate or wellbore condition, but also about how much of the earlier thermal disturbance remains and where it sits radially. In that sense, the well is not merely tested after drilling. It is tested while still thermally responding to drilling.

This perspective has several implications. The first is interpretive continuity. A circulation model, a shut-in correction, and a flow interpretation can be linked through one reduced state rather than being treated as separate calculations with only weak coupling. The second is greater discipline in parameter updating. Operational transitions should not license unrestricted recalibration of the model, because some parameters represent persistent properties of the thermal recovery process. The third is a more active role for temperature in flow diagnostics. Temperature is often used qualitatively to corroborate flow interpretation or detect entry, but the present formulation makes it a direct contributor to quantitative estimation of entering enthalpy and early rate when thermal disturbance is significant [33].

At the same time, the framework is not without limitations. The reduction to a finite set of radial thermal modes assumes that the operationally relevant disturbance can be represented compactly. This is likely reasonable in many test intervals, but more complex completion geometries, strong vertical heterogeneity, or pronounced multiphase effects could require richer representations. The entering-temperature weighting is also simplified relative to what a fully compositional or multiphase simulator would provide. In wells where phase change, Joule-Thomson effects, or strong thermal nonlinearity dominate, additional physical structure would be needed. The present model is therefore best understood as an intermediate-resolution inverse framework: more physical than generic online learning, more adaptive and data-assimilative than a fixed analytical correction, but not a substitute for detailed high-fidelity simulation in every case.

Another practical issue is data quality. The identifiability analysis makes clear that some latent directions are only weakly observed unless temperature, pressure, and rate are all available with reasonable fidelity. If bottom-hole temperature is sparse or noisy, the distinction between a cooler near-wellbore zone and a stronger wellbore heat-loss mechanism can become difficult to resolve. Likewise, if rate data are delayed or heavily filtered, inflow-weighting uncertainty increases. The proposed regularization and smoothing alleviate these issues, but they do not eliminate them. Careful use of prior information and realistic uncertainty quantification remain necessary.

The model also highlights an operational trade-off between responsiveness and physical conservatism [34]. Around a rate change or the onset of flow, a strong innovation in the temperature signal may encourage the estimator to adapt quickly. Yet too much agility can destroy consistency with the preexisting thermal state. The regularized recursive design adopted here explicitly favors conservative adaptation in the thermally sensitive directions of the model. In a field setting this is often desirable. A slowly corrected but physically plausible interpretation is usually more valuable than a rapidly updated but thermally inconsistent one. Still, the correct balance will depend on the intended use. If the model is being used for immediate operational decisions, more responsiveness may be needed. If the goal is post-test diagnosis, stronger smoothing and consistency constraints may be acceptable.

A final methodological point is that the reduced inverse model provides a natural bridge between analytical understanding and sequential estimation. The radial thermal basis and the dissipative structure preserve the intuition developed in classical heat-diffusion analysis. The recursive update, mode blending, and parameter regularization provide a way to use actual measurements as they arrive without abandoning that physical structure. This combination is what makes the framework different from both

simple correction formulas and generic sequence learning methods [35]. The former often lack adaptability to complicated operational histories. The latter often lack physically meaningful state variables and therefore struggle to preserve historical consistency in a system whose present observables depend strongly on earlier thermal forcing.

9. Conclusion

This paper presented an online inverse modeling framework for estimating transient near-wellbore thermal recovery and using that estimate to improve early multirate drill stem test interpretation after drilling circulation. The framework treated the disturbed formation temperature field as a latent radial-axial state coupled to wellbore thermal transport, compressible inflow, and operation-dependent observation maps. By doing so, it brought circulation, shut-in recovery, early inflow, and multirate test response into a single sequential estimation structure rather than handling them as disconnected stages.

The development proceeded by deriving the governing thermal and flow equations, introducing a reduced basis for the near-wellbore temperature disturbance and wellbore temperature field, defining an entering-temperature functional that links the radial thermal state to observed wellbore response, and formulating a recursive state and parameter estimator with mode-dependent operators and regularized updates. The paper further analyzed identifiability and stability, emphasizing the importance of preserving dissipative thermal structure and avoiding aggressive parameter changes in physically sensitive directions. A field-oriented validation strategy was then outlined, focusing on entering-temperature reconstruction, early-time rate inference, operational-transition adaptation, and uncertainty behavior under realistic data limitations. The central conclusion is that early drill stem test interpretation in circulated wells is best viewed as a history-dependent inverse problem. The relevant thermal state is not the undisturbed formation temperature, but the evolving near-wellbore disturbance left by prior circulation and modified by shut-in and flow. If that state is ignored, early inflow temperature and rate estimates can be biased even when pressure interpretation appears satisfactory. A reduced-order online inverse model provides a principled way to carry thermal history forward and use new measurements to refine it. Such a framework offers a coherent basis for further work on more detailed multiphase thermodynamics, richer completion geometries, and field deployment with streaming downhole measurements [36].

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